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Hybrid Approach for Damage Detection in Flexible Structures

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Recently, various system identification approaches have been developed and applied for the detection of damage in flexible structures. In this paper, modal characteristics extracted from vibration tests are used with an original finite element model in an identification approach developed to combine the advantages of two classes of techniques: eigensensitivity and multiple-constraint matrix adjustment. Here, physical property parameters are adjusted, as with eigensensitivity techniques, but model matrix characteristics are employed, as with matrix adjustment methods. The performance of this hybrid technique is shown with its application to data from a flexible-truss laboratory experiment.

Introduction

THE ability to locate and assess damage in flexible structures is becoming increasingly important for improving the performance and life of these systems. Many focused (or local) approaches have been developed and evaluated for the purpose of damage location and assessment, including X-ray, optical, infrared, and ultrasonic methods. Among the system (or global) methods currently in development, those that use vibration response and system identification have progressed considerably in the last five years.[1–6] Here, system identification refers to the process of obtaining an updated model to match the measured response. The motivation behind a system identification approach for damage detection is to quantify the damage information contained in the response as much and as effectively as possible.

In this work, damage to the structure refers to localized failure of a part of the structure. This failure can be a complete loss of capability in the part or a degradation of properties to an unacceptable level. We assume this failure or degradation would primarily affect the stiffness properties and therefore the modal characteristics of the dynamic response of the structure. This work is focused on damage in truss structures, so the algorithms that follow are applied to locate and assess the total or partial loss of stiffness in single or multiple members of a truss. However, these algorithms are not restricted to truss applications by design and so are applicable to other structures as well.

In general, a predamage finite element method (FEM) model is constructed and possibly adjusted to match the response of the undamaged structure. The inertia properties of this model are assumed to be essentially consistent with those of the damaged structure defined above.

Recent researchers have adopted a two-step damage detection process: first damage location, then assessment. For the first step, the use of "damage vectors," as presented by Zimmerman and Kaouk,[5] or "residual force vectors," as presented by Ricles and Kosmatka,[7] can locate damaged regions of the structure as represented in the FEM model. The second step is to assess the damage by algorithmically comparing the modal characteristics of the predamage FEM model to the postdamage structural response, producing an adjusted FEM model with adjustments limited to the region or regions defined in the first step.

In this paper, a method is presented for the damage assessment step that uses the modal characteristics extracted from a vibration test in a strategy developed to combine the advantages of two widely used system identification approaches: eigensensitivity and multiple-constraint matrix adjustment. In this hybrid technique, physical parameters are adjusted to reflect the FEM property matrix adjustments required to match the response characteristics experimentally determined for the structure. The performance of this method is shown with its application to data from a flexible-truss laboratory experiment. A discussion of the performance with respect to that of the two underlying system identification approaches is included, along with a presentation of practical considerations, such as for the incomplete measurement problem when the number of sensor outputs is less than the number of degrees of freedom (DOF) of the system.

Background

Approaches that employ parameter sensitivities and multiple-constraint matrix adjustment have shown promise for practical application in model refinement and damage detection scenarios. Sensitivity derivatives are widely used for design optimization.[8] Recently, applications to model refinement and damage detection in flexible structures have been investigated, often based on eigenvalue and eigenvector derivatives. Hendricks et al.[9] presented an eigenvalue sensitivity identification procedure and performed numerical simulations with an example structure designed to provide the low-frequency and clustered vibration modes characteristic of large flexible space structures. Property matrices (mass, damping, and stiffness matrices) were constructed in terms of a small set of physical property parameters. Estimations of corrections to initial parameters were determined by using first-order eigenvalue derivatives and the difference between measured and predicted frequencies. Intuitively, eigenvalue sensitivity methods are expected to be less effective than eigenvector sensitivity methods, since the former involve less information about the system. Flanigan[10] employed an eigenvector sensitivity approach for model refinement of a truss structure. Ricles and Kosmatka[7] illustrated the use of the eigenvector sensitivity approach for damage detection in elastic structures. First, residual modal force vectors were used to locate damage, after which a weighted eigenvector sensitivity analysis was carried out to assess the extent of the damage. Note that the incomplete measurement situation was addressed for damage assessment, but not for damage location or definition of a group of physical parameters.[11]

An advantage of sensitivity identification is that structure load paths are automatically preserved through the FEM model assembly using the physical parameters. Also, areas in a property matrix away from damage or refinement remain unaffected by using a localized set of physical parameters. Furthermore, sensitivity algorithms can be used to assess damage with incomplete measurements. A disadvantage is that an inclusive set of parameters must be defined before a sensitivity analysis can be successful. Moreover, most eigensensitivity algorithms are restricted to detection of small parameter changes due to truncation of higher order derivatives in the formulation. The nonlinearity and nonunimodal qualities of the eigenvalue or eigenvector functions are not compatible with the linearizing formulation, so that assessment of large changes, which might be expected

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with damage, is precluded in many cases. Experiments reveal these algorithms are often not robust with respect to errors in the modal data. Finally, eigensensitivity formulations sometimes lead to numerical difficulties with singular or near-singular sets of equations, in part complicated by the fact that the physical parameter set must be selected to encompass the subset required to represent the model changes.

Recently, modification of the classical eigensensitivity approach has improved its performance. Lin[12] proposed an improved eigensensitivity method that employs both analytical and experimental modal data (both eigenvalues and eigenvectors) to calculate sensitivity coefficients. Verification with a numerical experiment for updating a truss structure indicated the difficulties of small error (or damage) magnitude and slow convergence are overcome.

Although many commercially available model refinement codes use a sensitivity approach, applications of multiple-constraint matrix adjustment identification methods (also sometimes referred to as optimal-update methods) have recently shown promise for damage location and assessment in flexible structures. A detailed review of these methods can be found in Ref. 13. This approach produces, through the solution of a constrained optimization problem formulated with the matrix Frobenius norm,[13–17] adjusted FEM property matrices that more closely match the structural modal properties determined from tests of the structure. In various formulations, matrix characteristics such as the definiteness and the zero/nonzero pattern are imposed. For damage location and detection, this approach is used to establish areas of stiffness loss in the FEM model. Comparing the two stiffness matrices for a structure—one from the model of the undamaged structure and the other identified from limited measurement of the structure response—leads to the location of changed and therefore possibly damaged areas.

The applicability of matrix adjustment algorithms for damage location has been verified. Experience also suggests that these algorithms are not sensitive to testing errors that appear as inconsistencies in the extracted modal data. However, their application for damage assessment is limited by the matrix Frobenius norm used, which tends to spread changes throughout the entire matrix, leading to an inaccurate assessment. Damage, as we have defined it, is a localized effect. Moreover, it is difficult to interpret the physical meaning of the updated matrix elements that result.

The motivation of this research was to combine the advantages of both eigensensitivity and matrix adjustment techniques to create a hybrid approach for better detection of damage in flexible structures and particularly with inconsistent, real test data.

New Developments

To address the shortcomings of the eigensensitivity approach for damage location and assessment and to address the shortcomings of the matrix adjustment approach for damage assessment, a hybrid approach for assessment has been developed. Multiple-constraint stiffness matrix adjustment algorithms that allow for inconsistent data are formulated in general from the objective function to minimize the Frobenius norm of the matrix of residual force vectors[17]

$$\min \|\boldsymbol{K}(\boldsymbol{p})\boldsymbol{X} - \boldsymbol{M}\boldsymbol{X}\boldsymbol{\Omega}^2\|_F^2 \tag{1}$$

where $\boldsymbol{K}(\boldsymbol{p})$ is the $n \times n$ adjusted stiffness matrix, a function of physical parameters; $\boldsymbol{M}$ is the $n \times n$ mass matrix; $\boldsymbol{\Omega}$ and $\boldsymbol{X}$ are, respectively, the $n_m \times n_m$ diagonal matrix containing n_m measured circular frequencies and the $n \times n_m$ matrix containing n_m corresponding mode shapes, where the measured modes can be arranged in any order; n is the number of DOF in the FEM model; and n_m is the number of measured modes. For the hybrid approach, physical parameter sensitivities are introduced through a first-order expansion of the stiffness matrix

$$\boldsymbol{K}(\boldsymbol{p}) \approx \boldsymbol{K}(\boldsymbol{p}_0) + \sum_{j=i}^{n_p} \frac{\partial \boldsymbol{K}(\boldsymbol{p}_0)}{\partial p_j}\delta p_j \tag{2}$$

where $\boldsymbol{p}$ is the $n_p \times 1$ vector of the physical parameters, $\boldsymbol{p}_0$ is the vector of initial physical parameters, δp_j is the change in the jth physical parameter, and n_p is the number of physical parameters.

The Appendix presents the details of the derivation. A least-squares problem is the result, producing adjustments for the set of physical property parameters. However, a more straightforward derivation of the final least-squares inverse problem is possible if one considers the matrix eigenproblem to be a function of the defined set of physical parameters.

Start with the force balance of the undamped eigenproblem as

$$\boldsymbol{M}\omega_i^2\boldsymbol{x}_i = \boldsymbol{K}(\boldsymbol{p})\boldsymbol{x}_i, \qquad i = 1, 2, \ldots, n_m \tag{3}$$

where ω_i and $\boldsymbol{x}_i$ are, respectively, the measured circular frequency and the corresponding $n \times 1$ mode shape vector for the ith measured mode. Here $\boldsymbol{x}_i$ is full, a complete representation of the mode shape for the DOF corresponding to the FEM model. Therefore, if only partial measurements are available from test, an estimate of the full mode shape vector is determined using an expansion technique such as the one presented in Ref. 11.

A first-order Taylor series is employed to expand the function on the right-hand side of Eq. (3) so that we have

$$\boldsymbol{M}\omega_i^2\boldsymbol{x}_i \approx \boldsymbol{K}(\boldsymbol{p}_0)\boldsymbol{x}_i + \frac{\partial[\boldsymbol{K}(\boldsymbol{p}_0)\boldsymbol{x}_i]}{\partial \boldsymbol{p}}\delta\boldsymbol{p}, \qquad i = 1, 2, \ldots, n_m \tag{4}$$

The resulting set of algebraic equations can be combined in the form

$$\boldsymbol{A}\,\delta\boldsymbol{p} = -\boldsymbol{d} \tag{5}$$

where

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_1 \\ \boldsymbol{A}_2 \\ \vdots \\ \boldsymbol{A}_{n_m} \end{bmatrix}, \qquad \boldsymbol{d} = \begin{Bmatrix} \boldsymbol{d}_1 \\ \boldsymbol{d}_2 \\ \vdots \\ \boldsymbol{d}_{n_m} \end{Bmatrix} \tag{5a}$$

in which

$$\boldsymbol{A}_i = \frac{\partial[\boldsymbol{K}(\boldsymbol{p}_0)\boldsymbol{x}_i]}{\partial \boldsymbol{p}} \tag{5b}$$

$$\boldsymbol{d}_i = \left[\boldsymbol{K}(\boldsymbol{p}_0) - \boldsymbol{M}\omega_i^2\right]\boldsymbol{x}_i, \qquad i = 1, 2, \ldots, n_m \tag{5c}$$

are, respectively, the $n \times n_p$ Jacobian matrices and the $n \times 1$ damage vectors[5] or residual forces.[7] Even though the matrix $\boldsymbol{A}$ is of dimension $(nn_m) \times n_p$, with nn_m usually greater than $2n_p$, it is often rank deficient.

The vector of parameter adjustments $\delta\boldsymbol{p}$ is determined by solving the rank-deficient least-squares problem using an approach based on the singular value decomposition. Generally speaking, the adjustment is an iterative process:

1) The term $\boldsymbol{p}_0$ is defined from the initial FEM model.

2) The terms $\boldsymbol{A}$ and $\boldsymbol{d}$ are defined from the initial FEM model and experimental eigendata as specified in the equations above.

3) The parameters are updated by first solving Eq. (5) for $\delta\boldsymbol{p}$, then computing

$$\boldsymbol{p} = \boldsymbol{p}_0 + \delta\boldsymbol{p}$$

4) The updated $\boldsymbol{p}$ becomes the new initial parameter vector for the next iteration.

5) Procedures 2–4 are repeated until final convergence, which has been defined in this work as all $\delta p_i/p_i$ $(i = 1, 2, \ldots, n_p)$ are less than 0.01%.

If the relationships between the stiffness components and the physical parameters are linear, this method theoretically converges in one step. In a practical situation with errors in the measured data and initial FEM model, an iterative solution is generally required. The convergence limit was arbitrarily defined but was found to be suitable for this work.

As noted previously, the Appendix presents details of a derivation of this method starting from a multiple-constraint matrix adjustment approach and incorporating physical parameter sensitivities. Consequently, we considered the resulting technique to be a hybrid of these two approaches. Reference 18 presents more detailed discussions and also includes a derivation of Eq. (5) using Newton's method. Understanding that the hybrid method results from several different, though related, initial formulations has improved our understanding of its performance.

Compared to eigensensitivity approaches, this algorithm is better suited to assess the significant stiffness changes that can occur with damage since $\boldsymbol{K}(\boldsymbol{p})\boldsymbol{x}_i$, the right-hand-side of Eq. (3), is a linear function of the physical parameters if $\boldsymbol{p}$ contains only parameters that appear as linear functions in $\boldsymbol{K}(\boldsymbol{p})$. This algorithm also converges more readily than algorithms based on frequency or mode shape sensitivity that are nonlinear functions of the physical parameters.

An important observation for practical purposes is that only the modal and structural data corresponding to the DOF to which $\boldsymbol{p}$ directly contributes can affect the identification of $\boldsymbol{p}$, so only certain measurements of the structure are necessary for a particular parameter correction. Upon application, these measured DOF can be expanded by setting the DOF outside the region of influence of the physical parameter to zero with no impact on the assessment result. If critical members are designated in advance, the mathematical characteristics of this approach can be used to design the sensor placement.

If significant changes occur in the mass matrix as well, Eq. (3) can be rearranged into

$$\boldsymbol{0} = -\boldsymbol{M}(\boldsymbol{p})\omega_i^2\boldsymbol{x}_i + \boldsymbol{K}(\boldsymbol{p})\boldsymbol{x}_i \tag{6}$$

and expanded in the same way to solve for adjustments to the physical parameters. The form of Eq. (3) can also be similarly altered to produce various algorithms to satisfy different requirements, such as to include damping, or to make use of only partial response information. Details of these are presented in Ref. 18.

Note that Farhat and Hemez[19] independently developed a method that includes Eq. (4) above as an intermediate point in its formulation. They solved the n_m vector equations iteratively and separately for the physical parameters in conjunction with an iterative estimation of the expanded modes.

In the following sections, some practical issues involved in damage location and assessment are presented using data from modal tests of a laboratory truss structure to demonstrate and evaluate the hybrid approach.

Practical Considerations

In practice, we must consider errors arising from FEM modeling and from testing, in which the testing errors may include those due to nonfunctioning sensors.

Based on the assumptions that the initial FEM model is sufficiently accurate, that the testing errors are primarily due to nonfunctioning sensors, and that the sensor errors remain constant throughout the testing process, one approach to address these testing errors might be to have the measurements (frequencies, mode shapes, or both) from the postdamage response adjusted as follows:

$$\boldsymbol{Y}_{\text{cm}} = \boldsymbol{Y}_{\text{dm}} - \Delta\boldsymbol{Y} \tag{7a}$$

in which $\boldsymbol{Y}_{\text{cm}}$, $\boldsymbol{Y}_{\text{dm}}$, and $\Delta\boldsymbol{Y}$ are $q \times r$ matrices of corrected measurements, postdamage measurements, and error correction, respectively; q is the number of sensor outputs; and r is the number of measured modes used. The correction $\Delta\boldsymbol{Y}$ is assumed to be

$$\Delta\boldsymbol{Y} = \boldsymbol{Y}_{\text{um}} - \boldsymbol{Y}_a \tag{7b}$$

in which $\boldsymbol{Y}_{\text{um}}$ is a $q \times r$ matrix of predamage measurements and $\boldsymbol{Y}_a$ is a $q \times r$ matrix of analytical modal data. Note that all mode shapes are orthonormalized with respect to the mass matrix (or reduced mass matrix if mode shapes are reduced) before being corrected using Eq. (7a). In the following, this will be referred to as global correction.

A second correction approach can be adopted once one considers that, when applying this hybrid algorithm, only adjacent DOF information will influence the identification of the physical parameter for a truss member. Thus only local mode shape error corrections are necessary.

Experimental Applications

The laboratory structure is a cantilevered eight-bay truss, constructed as part of the Dynamic Scale Model Technology (DSMT) Program at NASA Langley Research Center.[20] In Fig. 1 the FEM model mesh for this truss is presented, with the root (or fixed end) of the truss on the right. Note that the DOF are numbered from the free end to the root. Five typical damage cases are defined in Table 1 with respect to Kashangaki's labeling, including total loss of stiffness for individual and for multiple members of the truss. Frequency and mode shape information for the first five modes of the structure were extracted from measured acceleration time histories for the undamaged truss and for each of the five cases considered.[20] In these tests, accelerometers were located at each of the 96 DOF corresponding to the FEM model, allowing the future flexibility of selecting any subset of the measurements.

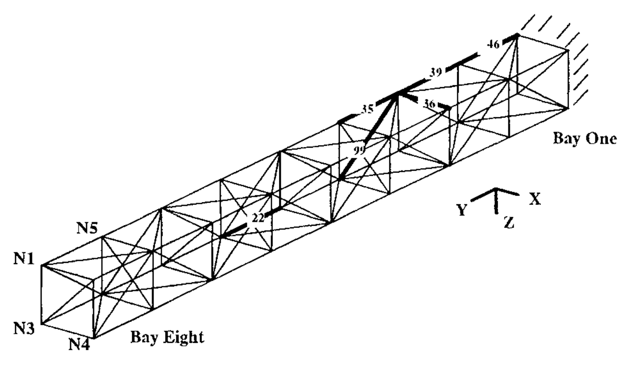
Recall that damage detection is a two-step process including location and assessment. In the first (location) step, a physical parameter set is defined. Differences between the damage vectors from predamage response and those from postdamage response were used to eliminate the force imbalance due to modeling and testing errors. A complete study of the damage vectors for this eight-bay truss was presented by Kaouk and Zimmerman.[21] As an example here, Figs. 2a–2c are, respectively, the predamage vector sums, the postdamage vector sums, and the difference for case 1. To improve confidence in the damage location indication, each figure is the sum of five damage vectors resulting from the five modes, instead of a damage vector from a single mode. Several peaks appear in both the pre- and postdamage vectors, indicating force imbalance due to both modeling errors and sensor output errors. It is generally believed that peaks at DOE 44 and 45, and possibly 85 and 88, are due to sensor output errors, whereas peaks at the fixed and free ends are mainly due to structure modeling errors including omission of shaker stinger stiffnesses. The plot of the difference between these pre- and postdamage vectors clearly shows a single distinct peak at DOF 86, indicating damage to the truss member corresponding to element label 46 (member 46). The damage location results for cases 2–5 can be found in Ref. 18.

The parameters contained in $\boldsymbol{p}$ here are defined as the stiffnesses of potentially damaged truss members. For each damage case, two sets of parameters were defined. Set 1 in each case contains parameters

Table 1 Eight-bay damage case definitions

Damage case	Element label	Element type	Bay
1 (a)[a]	46	Longeron	1
2 (h)	35	Longeron	3
3 (l)	22	Longeron	5
4 (e)	36	X-batten	3
5 (o)	35 and 99	Longeron and diagonal	3

[a]Labeling in parentheses from Kashangaki's designations (Ref. 20).



nodal joint cluster mass	0.4876 kg
member length (x, y, or z direction)	0.5 m
strut EA	$5.06\text{x}10^{-4}$ N
density ϱ_l (longeron)	$2.023\text{x}10^{-6}$ kg/m^3
density ϱ_d (diagonal)	$2.173\text{x}10^{-6}$ kg/m^3

Fig. 1 Eight-bay truss model and damage case labeling.

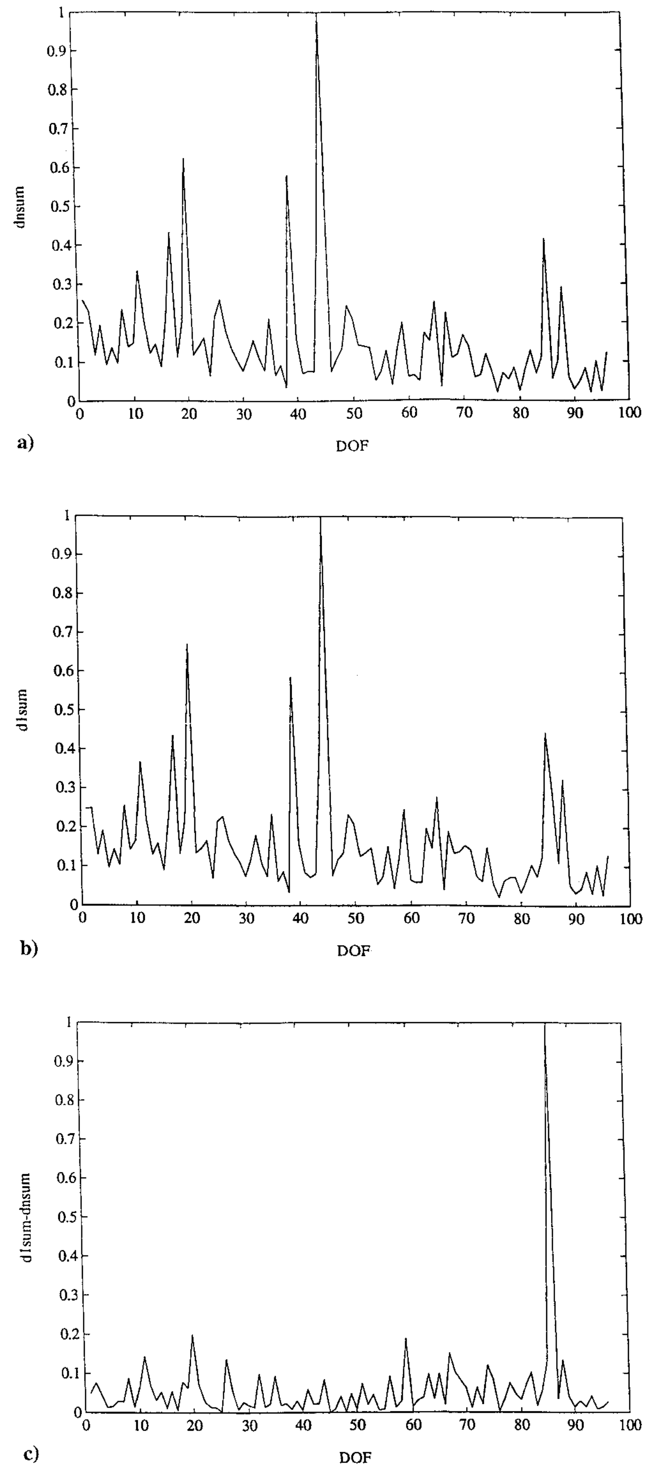


Fig. 2 Damage vector sum vs DOF for damage case 1: a) predamage, b) postdamage, and c) difference between pre- and postdamage.

indicated by examining only the postdamage vector. A common subset of 13 parameters representing initial model and sensor errors is included in each. These parameters are listed in Table 2 and would be the set of parameters defined if one considered only the predamage vector. In each case, set 2 includes only parameters clearly defined in the difference between the post- and predamage vectors. In applying this method, set 2 would typically be the only set defined, but set 1 is included here to illustrate issues and performance.

The initial value and corrections of $\boldsymbol{p}$ for case 1 are shown in Table 3, where $\boldsymbol{p}_0$ is the initial value of $\boldsymbol{p}$; $\boldsymbol{p}_u$ and $\boldsymbol{p}_d$ are, respectively, the modified values of $\boldsymbol{p}$ using pre- and postdamage measurements; and $\boldsymbol{p}_{cd}$ is the updated value of $\boldsymbol{p}$ employing the postdamage measurements after global corrections. For set 1 we see

Table 2 Definitions of parameters from predamage vector sum

Parameter	Node 1	Node 2	Bay
p_1	1	6	8
p_2	3	7	8
p_3	4	7	8
p_4	6	9	7
p_5	9	15	6
p_6	13	15	6
p_7	1	2	8
p_8	15	17	5
p_9	17	21	4
p_{10}	17	22	4
p_{11}	29	30	2
p_{12}	4	6	8
p_{13}	23	24	4

that the adjustments to the first 13 parameters, which correspond to force imbalance with the predamage vectors, are nearly identical for both $\boldsymbol{p}_u$ and $\boldsymbol{p}_d$, most of which are quite different from their corresponding initial values. The column of the difference $\boldsymbol{p}_d - \boldsymbol{p}_u$ shows values near zero except for parameter 14, which corresponds to the damaged member. This again indicates that these parameters are involved with initial modeling and testing errors rather than with the damage. Some parameter adjustments from errors related to testing are avoided with global measurement corrections, as is reflected in $\boldsymbol{p}_{cd}$ (parameters 3, 8, and 10). However, assessments of p_{14} using both parameter sets produce similar satisfactory results, showing near 100% loss of stiffness of member 46, verifying this new algorithm, and demonstrating the characteristic of local identification with local information using this algorithm.

As indicated by damage vectors, both cases 2 and 3 involve significant sensor errors adjacent to or near the damaged member, which affects the assessment of damage with direct use of measurements. In Table 4, adjustments to parameter p_9, whose corresponding truss member is adjacent to the damaged member (element label 35), were indicated when the pure postdamage response was used. After global or local (around p_9 and p_{14}) corrections of the modal data, the damaged member is still identified in $\boldsymbol{p}_{cd}$ with a complete stiffness loss, whereas the nearby element associated with p_9 is not. The outcomes in Table 5 reflect that selected parameters involving testing or modeling errors affect the accuracy of assessment of real damage when adjacent to the damaged member. This situation is improved by measurement corrections.

With the selection of the large parameter set (set 1) and the identification of nonphysical adjustments in these parameters, confidence in the approach is diminished. However, similar nonphysical identification results also occur when the eigensensitivity approach or matrix adjustment approaches are employed for this truss; these have been attributed to unmodeled fixed-end and test apparatus conditions. In the hybrid approach, the independence of parameter adjustments due to the local-influence characteristic allows confidence in the damage assessment even though other parameter changes away from that area are nonphysical. Therefore, each indication must be evaluated individually if care is not taken in the initial parameter set selection.

If the number of measured locations is less than the system dimension and the damaged parts are within the measured regions, the reduced mode shapes can be augmented by filling the unmeasured parts with zeros. Table 6 shows the evaluation results of cases 1–3 when only set 2 and the measured mode shape locations adjacent to each damaged member were used. These results demonstrate that such reduction of the measurements does not affect the assessment. However, this mode shape expansion technique is not able to indicate damage outside the measured regions. In this case, Smith and Beattie's expansion method[11] can be adopted.

Case 4 is difficult to assess correctly because of relatively trivial deformations of the X-batten. Neither the eigensensitivity nor the matrix adjustment approach is able to correctly locate and assess the damage. The hybrid method did not improve on this. For the multiple-damage case (case 5), the new algorithm had a similar performance as it did in case 2 with no additional difficulties. The pa-

Table 3 Damage assessment results using hybrid method for case 1

	Parameter set 1, $\times 1.75 \times 10^6$ N/m				Parameter set 2, $\times 1.75 \times 10^6$ N/m		
p_0	p_d	p_u	p_{cd}	$p_d - p_u$	p_0	p_d	p_{cd}
0.457	−0.461	−0.442	−0.096	−0.019			
1.291	−0.710	−0.777	−0.633	0.066			
0.457	−0.468	−0.436	0.407	−0.031			
0.457	1.042	1.142	0.909	−0.100			
0.457	0.036	−0.018	0.129	0.054			
1.291	0.117	0.119	−0.218	−0.003			
1.291	0.190	0.203	−0.134	−0.013			
0.457	−0.208	−0.179	0.446	−0.029			
1.291	1.220	1.308	1.190	−0.089			
0.457	−0.390	−0.408	0.468	0.018			
1.291	−0.225	−0.208	−0.896	−0.019			
0.457	−0.154	−0.164	0.796	0.010			
1.291	−0.670	−0.614	−0.962	−0.056			
1.291[a]	−0.040	0.976	−0.094	−1.017	1.291	−0.040	0.082

[a]The term p_{14} corresponds to element label 46.

Table 4 Damage assessment results using hybrid method for case 2

	Parameter set 1, $\times 1.75 \times 10^6$ N/m				Parameter set 2, $\times 1.75 \times 10^6$ N/m		
p_0	p_d	p_u	p_{cd}	$p_d - p_u$	p_0	p_d	p_{cd}
0.457	−0.445	−0.442	−0.462	−0.003			
1.291	−0.701	−0.777	−2.086	0.076			
0.457	−0.399	−0.436	−0.077	0.038			
0.457	1.151	1.142	1.426	0.009			
0.457	−0.103	−0.018	0.059	−0.085			
1.291	0.176	0.119	0.068	0.056			
1.291	0.199	0.203	0.380	−0.004			
0.457	−0.248	−0.181	−0.181	−0.068			
1.291	−0.436[b]	1.290	1.573	−1.726			
0.457	−0.291	−0.407	−0.572	0.116			
1.291	−0.205	−0.208	−0.113	0.003			
0.457	−0.188	−0.164	−0.155	−0.024			
1.291	−0.428	−0.614	−0.520	0.185			
1.291[a]	−0.021	1.246	0.014	−1.267	1.291	−0.024	−0.024

[a]The term p_{14} corresponds to element label 35.
[b]Involves testing errors.

Table 5 Damage assessment results using hybrid method for case 3

	Parameter set 1, $\times 1.75 \times 10^6$ N/m				Parameter set 2, $\times 1.76 \times 10^6$ N/m		
p_0	p_d	p_u	p_{cd}	$p_d - p_u$	p_0	p_d	p_{cd}
0.457	−0.420	−0.450	−0.194	0.030			
1.291	−0.914	−0.792	1.004	−0.121			
0.457	−0.302	−0.406	−0.142	0.104			
0.457	1.797	2.227	1.638	−0.431			
0.457	−0.051	−0.007	−0.020	−0.044			
1.291	0.113	0.104	0.034	0.009			
1.291	0.140	0.208	0.285	−0.068			
0.457	−0.122	−0.155	0.015	0.033			
1.291	1.794	1.347	1.317	0.448			
0.457	−0.361	−0.406	−0.098	0.045			
1.291	−0.161	−0.208	−0.283	0.047			
0.457	−0.270	−0.256	0.533	−0.015			
1.291	−0.626	−0.614	−0.121	−0.012			
1.291[a]	0.283	8.093[b]	0.015	−7.811	1.291	−0.398	−0.591

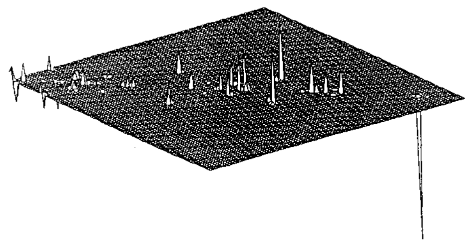
[a]The term p_{14} corresponds to element label 22.
[b]Involves testing errors.

rameters for the two damaged members (35 and 99) were identified using set 2 to have 94.9% and 98.2% stiffness loss, respectively.[18] These five cases were selected to present representative results, with examples of more straightforward (case 1) and more difficult (cases 3–5) applications.

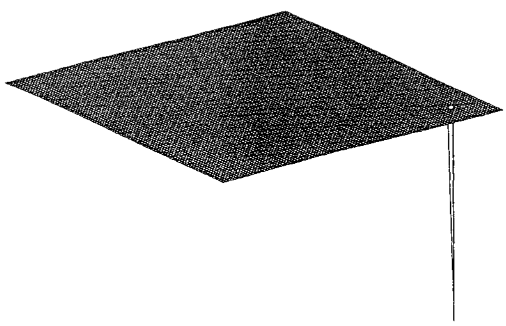
For comparison with the underlying techniques of the hybrid method, the updated results of case 1 using matrix adjustment and eigenvector sensitivity techniques are also presented. When applying the matrix adjustment approach,[22] a sparsity pattern was imposed on the updated stiffness matrix at each iteration to allow only those elements in the potentially damaged areas of the updated stiffness matrix to be modified. This was done by conducting an element-by-element multiply of the updated stiffness matrix with a 96×96 sparsity matrix in which the matrix entries related to the defined parameters were set to 1 whereas the others were set to zero. The two parameter sets (1 and 2) established above were used to produce two respective sparsity matrices (sparsity 1 and sparsity 2). The results of applying a multiple-constraint matrix adjustment to case 1 show error peaks in mesh plots of the difference between the updated and initial stiffness matrices. Pre- and postdamage measurements were used in turn, and the sparsity 1 pattern was employed. Damage shows up distinctively in the mesh of the difference of the final updated stiffness matrices,[18,22] similar to the results using the hybrid technique.

Table 6 Evaluation results of cases 1–3 using hybrid method, set 2, and local measurements

Case 1, 1.75×10^6 N/m		Case 2, 1.75×10^6 N/m		Case 3, 1.75×10^6 N/m	
p_0	p_d	p_0	p_d	p_0	p_d
1.2913	−0.1922	1.2913	−0.0241	1.2913	−0.3979



a) 1.1435 x1.75e6 N/m



b) 1.1444 x1.75e6 N/m

Fig. 3 Difference between adjusted stiffness matrices from pre- and postdamage measurements using matrix adjustment approach for case 1: a) sparsity pattern 1 and b) sparsity pattern 2.

Results with globally corrected postdamage measurements indicate that many errors are significantly reduced, whereas a few additional errors are introduced. The results of case 1 using sparsity 2 show the matrix adjustment algorithm converging more quickly than when using sparsity 1. The mesh-plot difference results and the stiffness loss magnitudes for these two examples are shown in Figs. 3a and 3b.

Compared to the performance of the multiple-constraint matrix adjustment technique, the hybrid method shows several advantages:

1) The hybrid method required an order of $\mathcal{O}(10^2)$ less computational effort for this application. The slowness of the iterative sparsity-preserving multiple-constraint matrix adjustment algorithms is attributed to having many matrix elements that are modified and then set to zero at each step in the process.

2) Only local information is needed for parameter assessments with the hybrid method. This property can also be used for guidance in both testing and damage identification.

3) The hybrid correction directly gives damage information for physical parameters without affecting the structure of the modified FEM matrix. Multiple-constraint matrix adjustment approaches produce matrix element changes that are at times more difficult to interpret.

An advantage of the multiple-constraint matrix adjustment approach is that convergence is ensured since the adjustment iterates between two convex sets. A proof is included in Ref. 18. Additional comments are in Ref. 22. Experience has also shown this matrix adjustment algorithm to perform equally well with various combinations of the measured modes and to have less sensitivity to eigendata errors than eigensensitivity applications. The hybrid approach has also demonstrated these qualities with respect to the measured eigendata.

On the other hand, the eigenvector sensitivity approach does not produce correct results for any of these five defined cases, presumably because the loss of stiffness of an individual member was too large in each case. An analytical simulation of this sensitivity method employing the parameters in case 1 reveals that it will work for a small stiffness loss, such as estimating 20% damage of member 46 by use of one parameter and measurements from the fifth mode.

Compared to the performance of the eigensensitivity approach, the hybrid method shows several advantages:

1) The hybrid method can evaluate a less restricted damage range. In this case greater than 20–25% stiffness loss can be identified with the hybrid approach, but not the eigensensitivity approach.

2) The number of parameters used is not limited. With the eigensensitivity approach, the larger of the two parameter sets could not be used with available data to produce a result.

3) No sensitivity to mode selection was observed for the hybrid method. The eigensensitivity approach either did not converge or converged to incorrect values with most mode combinations and errors as small as 1% introduced in numerical experiments.

4) The hybrid method generally requires less computational effort. The built-in requirement to recalculate eigendata from iteration to iteration makes the eigensensitivity algorithm more computationally intensive.

Eigensensitivity algorithms also require knowledge of which measurement is associated with which analytical mode, increasing the difficulties for situations involving closely spaced modes in particular. In contrast, the hybrid method was developed to address these shortcomings while assessing physical parameters and thus is better suited for application to damage situations.

Summary

The goal of this work was to start with a multiple-constraint matrix adjustment formulation and incorporate physical parameter sensitivities, so that the resulting method identifies changes in physical parameters rather than in matrix elements. This hybrid approach has some demonstrated advantages over sensitivity and multiple-constraint matrix adjustment techniques for damage assessment in flexible structures. Performance advantages of the approach and issues for practical application with sensor errors were shown with its application to data from a laboratory truss experiment.

Appendix

For

$$f = \|K(p)X - MX\Omega^2\|_F^2 \tag{A1}$$

then

$$f = \left\| \left[K(p)x_1\; K(p)x_2 \cdots K(p)x_{n_m}\right] - \left[Mx_1\omega_1^2\; Mx_2\omega_2^2 \cdots Mx_{n_m}\omega_{n_m}^2\right] \right\|_F^2 \tag{A2}$$

$$= \left\| r_1\, r_2 \cdots r_{n_m} \right\|_F^2 = \sum_{i=1}^{n_m} \|r_i\|_2^2 = \sum_{i=1}^{n_m} r_i^T r_i \tag{A3}$$

where

$$r_i = \left[K(p) - M\omega_i^2\right]x_i \tag{A4}$$

is referred to as a residual force vector.

Let

$$K(p) = K(p_0) + \sum_{j=1}^{n_p} \frac{\partial K(p_0)}{\partial p_j}\delta p_j \tag{A5}$$

Therefore

$$r_i = \left[K(p_0) - M\omega_i^2 + \sum_{j=1}^{n_p} \frac{\partial K(p_0)}{\partial p_j}\delta p_j\right]x_i = d_i + e_i \tag{A6}$$

in which

$$d_i = r_i(p_0) = \left[K(p_0) - M\omega_i^2\right]x_i \tag{A6a}$$

$$e_i = \sum_{j=1}^{n_p} \frac{\partial K(p_0)}{\partial p_j}\delta p_j x_i = \frac{\partial[K(p_0)x_i]}{\partial p}\delta p = A_i\,\delta p \tag{A6b}$$

are, respectively, the definitions of the damage vector and error vector.

If we set

$$\frac{\partial f}{\partial p} = \left[\frac{\partial f}{\partial p_1}\ \frac{\partial f}{\partial p_2}\ \cdots\ \frac{\partial f}{\partial p_{n_m}}\right] = [0\ 0\ \cdots\ 0] = 0^T \tag{A8}$$

then

$$2\sum_{i=1}^{n_m} r_i^T \frac{\partial r_i}{\partial p} = 2\sum_{i=1}^{n_m}(d_i + e_i)^T\frac{\partial e_i}{\partial p}$$

$$= 2\sum_{i=1}^{n_m}\left(d_i^T + e_i^T\right)A_i = 0^T$$

or

$$\sum_{i=1}^{n_m}\left(A_i^T d_i + A_i^T A_i\,\delta p\right) = 0 \tag{A9}$$

The preceding equation is equivalent to

$$A^T A\,\delta p = -A^T d \tag{A10}$$

which is Eq. (5) in the main text in the "normal equations" form of the least-squares problem.

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